

# Laplace transform properties

---

$$L\left[\frac{df(t)}{dt}\right] = sF(s) - f(0)$$

$$L\left[\frac{d^2 f(t)}{dt^2}\right] = s^2 F(s) - sf(0) - f'(0)$$

$$L\left[\frac{d^3 f(t)}{dt^3}\right] = s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$$

*general case*

$$L\left[\frac{d^n f(t)}{dt^n}\right] = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) \\ - \dots - f^{(n-1)}(0)$$

# Laplace transform with non-zero initial conditions

---

$$\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = \delta(t)$$

$$\dot{y}(0) = 1; y(0) = 1$$

# Laplace transform with non-zero initial conditions

---

$$\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = \delta(t)$$

$$\dot{y}(0) = 1; y(0) = 1$$

$$L[\ddot{y}(t)] + 5L[\dot{y}(t)] + 6L[y(t)] = L[\delta(t)]$$

$$[s^2 Y(s) - s \cdot y(0) - \dot{y}(0)] + 5L[sY(s) - y(0)] + 6Y(s) = 1$$

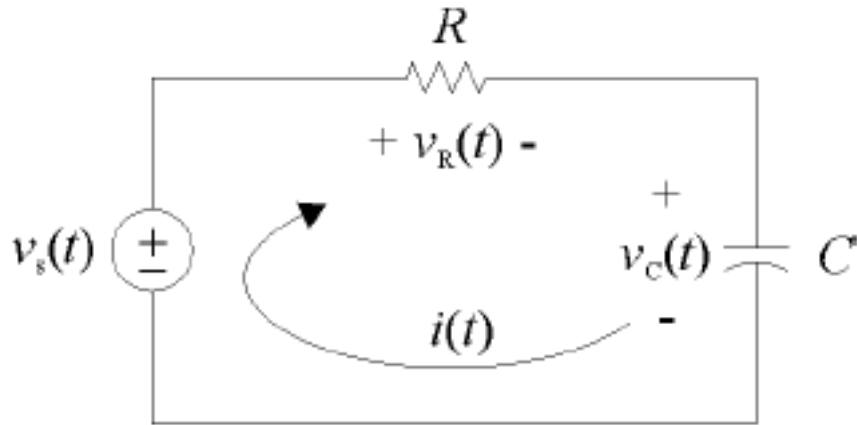
$$Y(s)(s^2 + 5s + 6) - s - 1 - 5 = 1$$

$$Y(s) = \frac{s + 7}{s^2 + 5s + 6}$$

$$Y(s) = \frac{5}{s + 2} + \frac{-4}{s + 3} \quad \Longrightarrow \quad y(t) = 5e^{-2t} - 4e^{-3t}$$

# Laplace transform with non-zero initial conditions

---



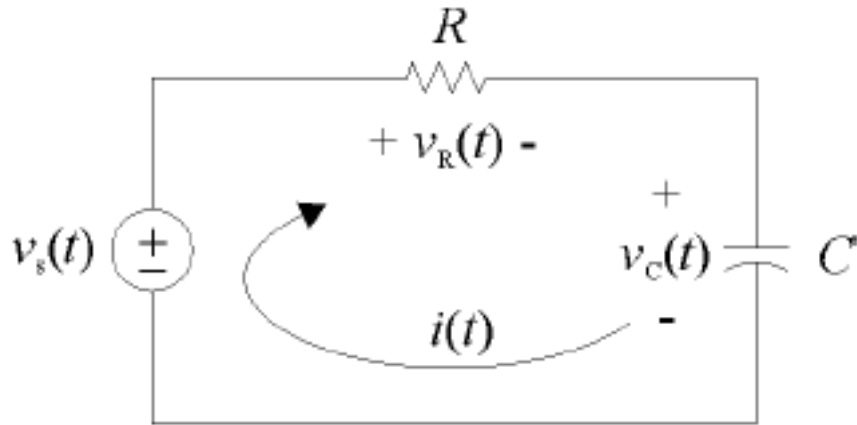
$$V_s(t) = RC \frac{dV_c(t)}{dt} + V_c(t)$$

$$V_s(t) = u(t)$$

$$V_c(0) = 0.2$$

# Laplace transform with non-zero initial conditions

---



$$V_s(t) = RC \frac{dV_c(t)}{dt} + V_c(t)$$

$$V_s(t) = u(t)$$

$$V_c(0) = 0.2$$

$$V_s(s) = RC(sV_c(s) - V_c(0)) + V_c(s)$$

$$V_c(s) = \frac{1}{1 + RCs} \cdot \left( \frac{1}{s} + 0.2RC \right)$$

$$V_c(t) = 1 - e^{-\frac{t}{RC}} + 0.2 \cdot e^{-\frac{t}{RC}} = 1 - 0.8 \cdot e^{-\frac{t}{RC}}$$

# Electrical safety in biomedical instrumentation



**COLUMBIA | ENGINEERING**  
The Fu Foundation School of Engineering and Applied Science



# Electrical safety of medical equipment

---

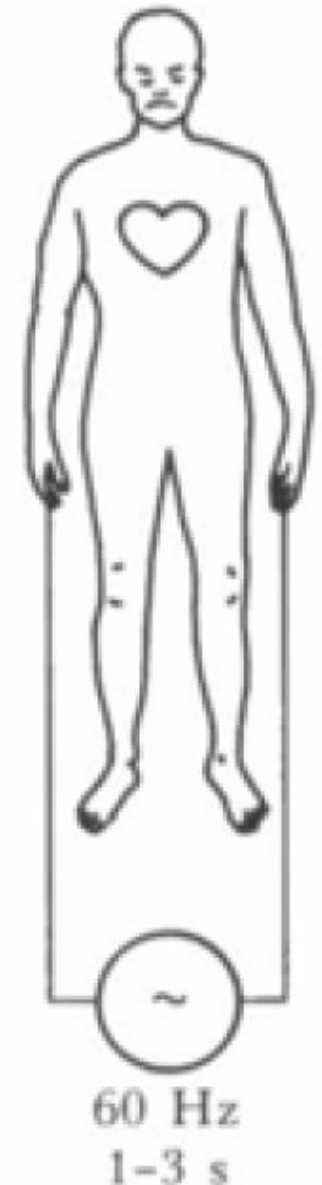
The requirements for the electrical safety of medical equipment are **much more stringent** than those for other electrical devices.

- Patient may be simultaneously connected to several medical devices or conductive therapeutic fluids.
- Medical devices may be in directly contact with internal tissue, e.g. through natural orifices or breaks in the skin.
- A situation that a healthy person could cope with may be disastrous to a weakened or hospitalized patient.
- Patient may be debilitated or immobilized and unable to respond to stimuli.

# Electrical safety

---

- Unlike other standards, electrical safety is not considered to be dependent on voltage, but on ***leakage currents***.
- For a physiological effect of electrical shocks to occur, the body must become part of an electric circuit. Current must enter the body at one point and leave at some other points.

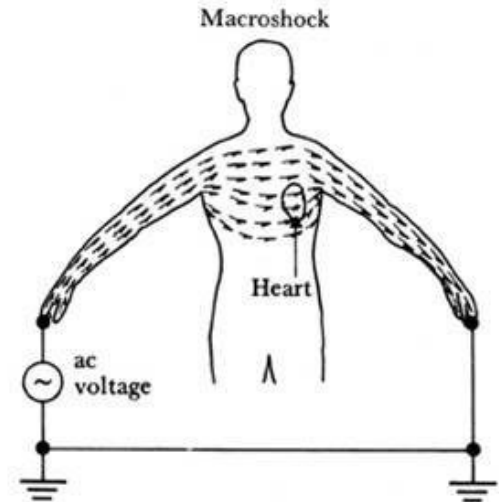




# Macro vs. Micro shock

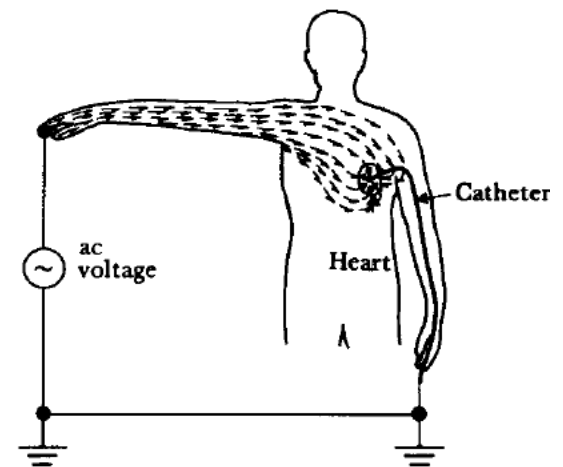
- **Macroshock**

- externally applied current spreads through the body
- The current is less concentrated



- **Microshock**

- applied current is concentrated at an invasive point
- generally only dangerous if current flows through the heart



# Electrical safety

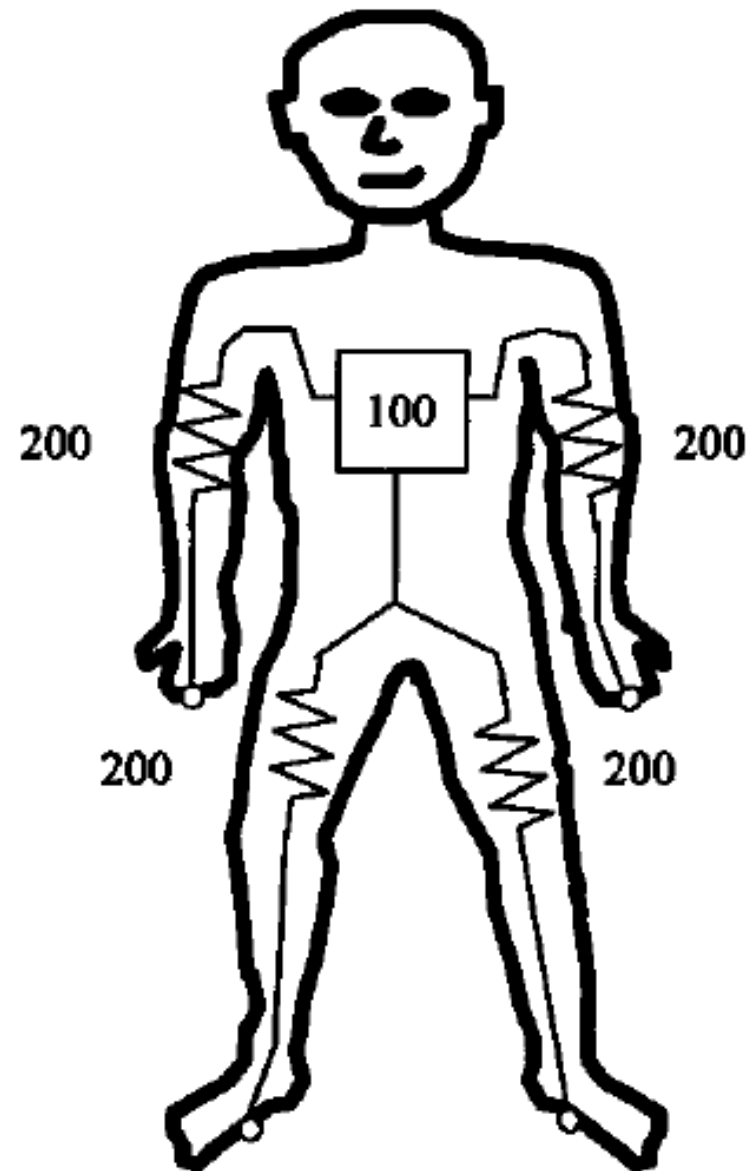
---

- Ohm's law: the magnitude of the current equals to the applied voltage divided by the sum of the series impedances of the body tissues and the two interfaces at the entry points.
- The largest impedance is often the skin resistance at the contact surface
  - Dry skin has a resistivity of 15 k $\Omega$
  - Wet skin has a resistivity of ~1% of its dry value

# Electrical safety

---

- Once the skin is bypassed, internal body tissue, which acts as a volume conductor, determines the resistance.
  - Internal body resistance is about 200  $\Omega$  for each limb
  - about 100  $\Omega$  for the trunk.



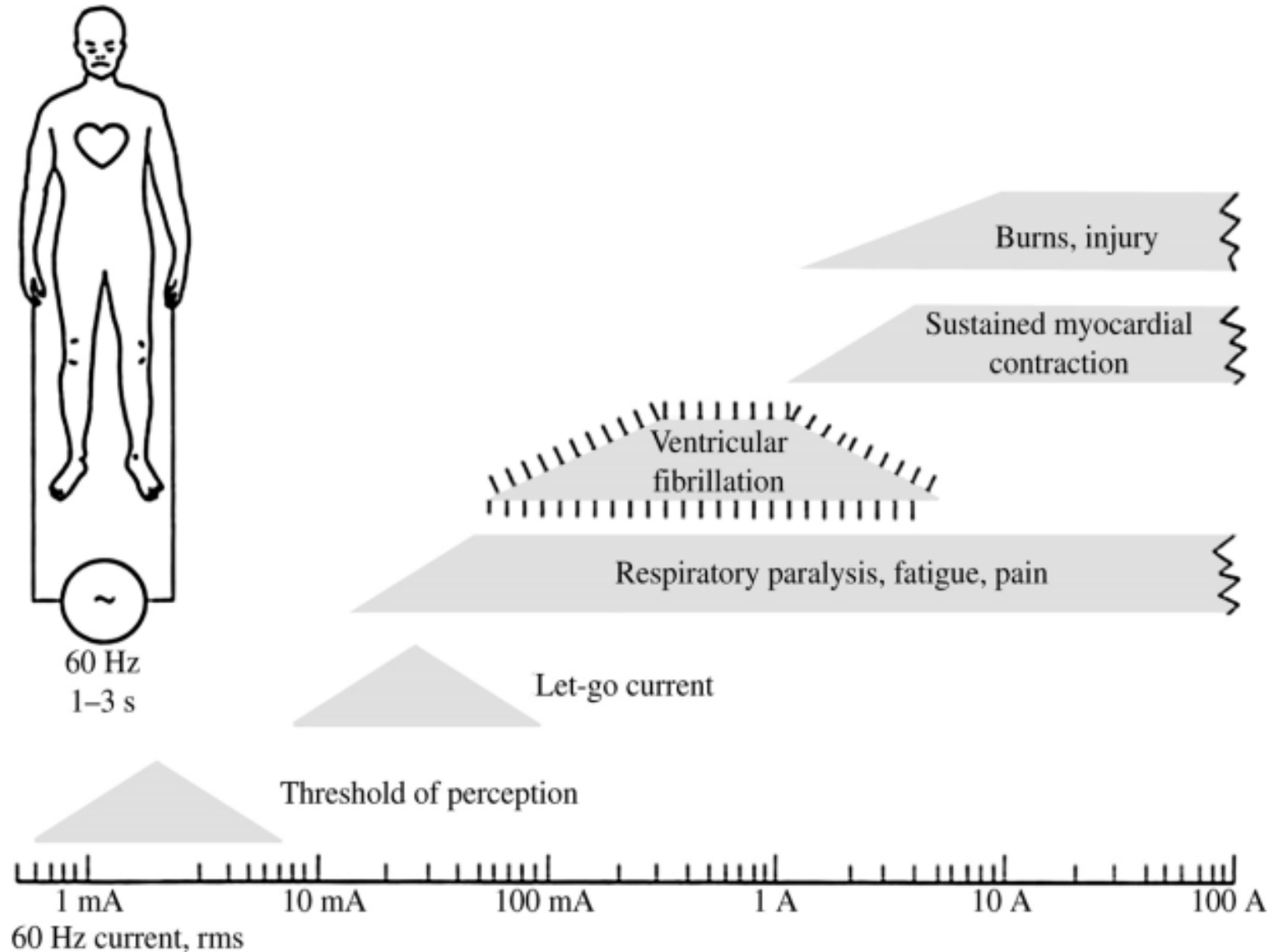
# Physiological effects of electrical current

---

## Physiological effects:

- Activation of excitable tissue (nerve and muscle.)
  - Brain
  - Heart: the most serious consequence that popularly happened
- Resistive heating of tissue.
- Electrochemical burns and tissue damage for direct high current.

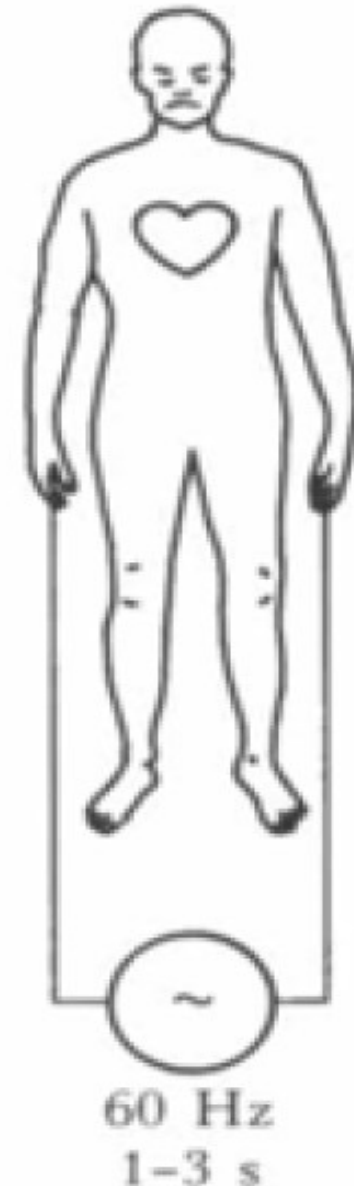
# Physiological effects of electrical current



# Threshold of perception ( $\sim 500 \mu\text{A}$ )

---

- Current at the threshold of perception is the minimal current that an individual can detect.
- Domestic and industrial appliances have a leakage current limit of  $500 \mu\text{A}$ ,

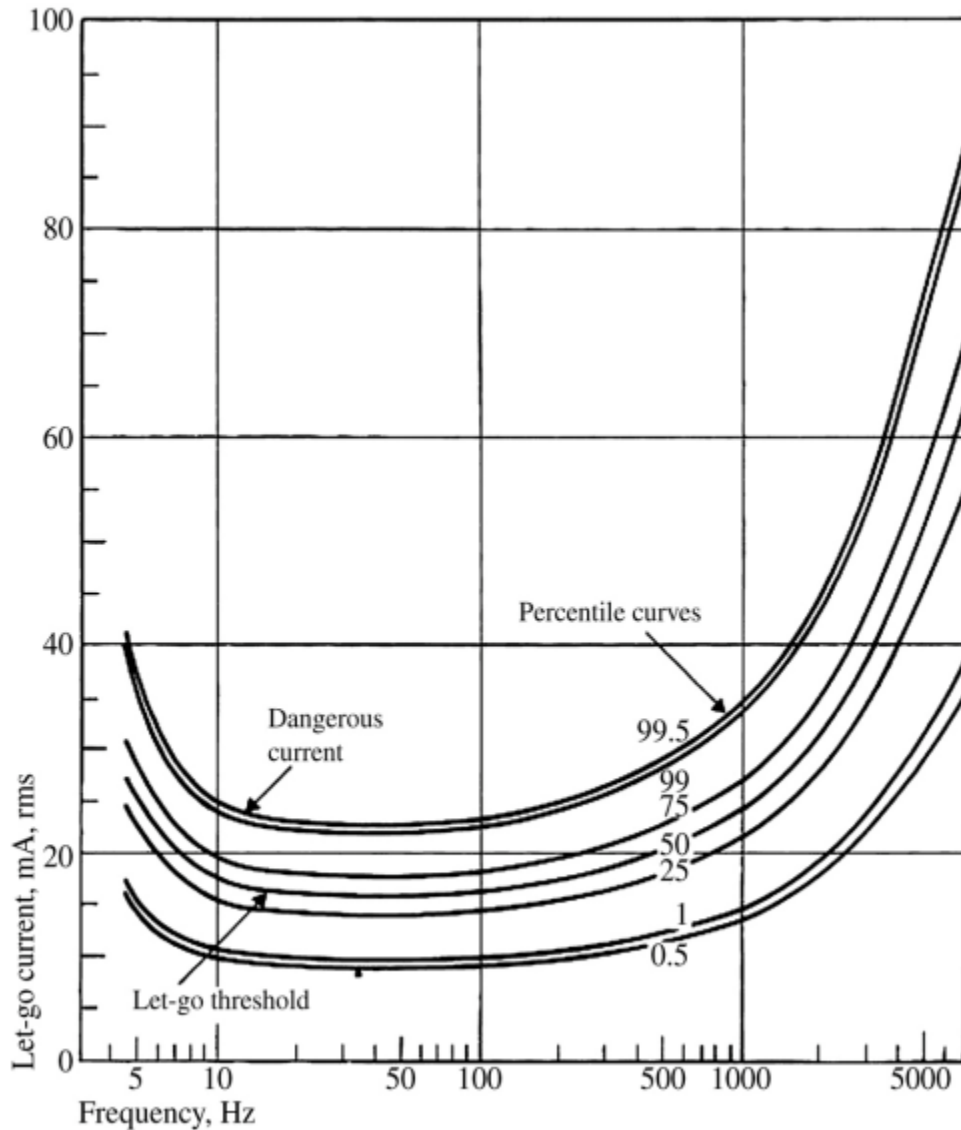


# Let-Go current (10 mA)

---

- As the current increases, involuntary contractions will happen for all forearm muscles. Since flexor muscles are stronger than extensor muscles, the subject may be unable to voluntarily release the grip.
- The let-go current is defined as the maximal current at which the subject can release an energized conductor grasped in the hand.
- Involuntary contractions of muscles or reflex withdrawals by a subject experiencing any current around threshold may also cause secondary physical injuries (e.g., being caused to fall off a ladder).

# Let-Go current



- Let-go current is dependent upon frequency
- Minimal let-go current occurs at 20-100 Hz, which covers commercial power-line frequencies of 50-60 Hz



# Respiratory paralysis, pain, and fatigue (10-20 mA)

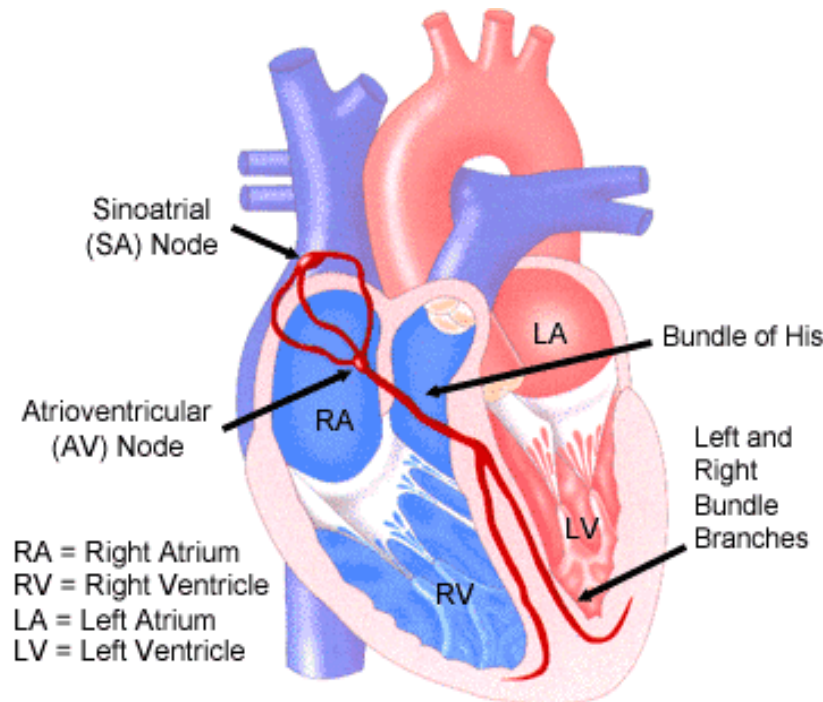
---

- Electrical currents can cause involuntary contraction of respiratory muscles severe enough to bring asphyxiation if the current is not interrupted.
- Involuntary contractions of the muscles caused by electrical stimulation and stimulation of the nerves can also be painful and cause fatigue if there is long exposure.

# Ventricular fibrillation (75-400 mA)

---

- The heart beats because of a small electrical current generated by a group of muscle cells in the walls of the heart.
- This group of cells work in a **coordinated** manner to generate rhythmic heart beats



# Ventricular fibrillation

---

Healthy rhythm



- Current passing through the chest also flows in part through the heart. If the magnitude of the current is sufficient to excite only part of the heart muscle, then the normal propagation of electric activity in the heart muscle is disrupted.

# Ventricular fibrillation (75-400 mA)

---

- If the cardiac electric activity is sufficiently disrupted, the heart rate can rise to 300 beats per minute as reentrant waveforms of depolarization randomly sweep over the ventricles.
- The pumping action of the heart stops and death occurs within minutes.

**Ventricular fibrillation**



# Ventricular fibrillation

---

- Shock duration
  - Fibrillation current is inversely proportional to the shock pulse duration, i.e. the longer pulses, the lower current needed to cause fibrillation.
- Body weight
  - Fibrillation current increases with body weight
  - 50 mA RMS for 6 Kg dogs
  - 130 mA RMS for 24 Kg dogs
  - Research using human subjects or animals must be approved an ethics committee

# Defibrillation

---

- Normal rhythmic activity returns only if a brief high-current pulse from a defibrillator is applied to depolarize all the cells in the heart simultaneously. After the cells relax together, a normal rhythm typically returns.

*Defibrillator*



# Burns and physical injury ( $> 1\text{ A}$ )

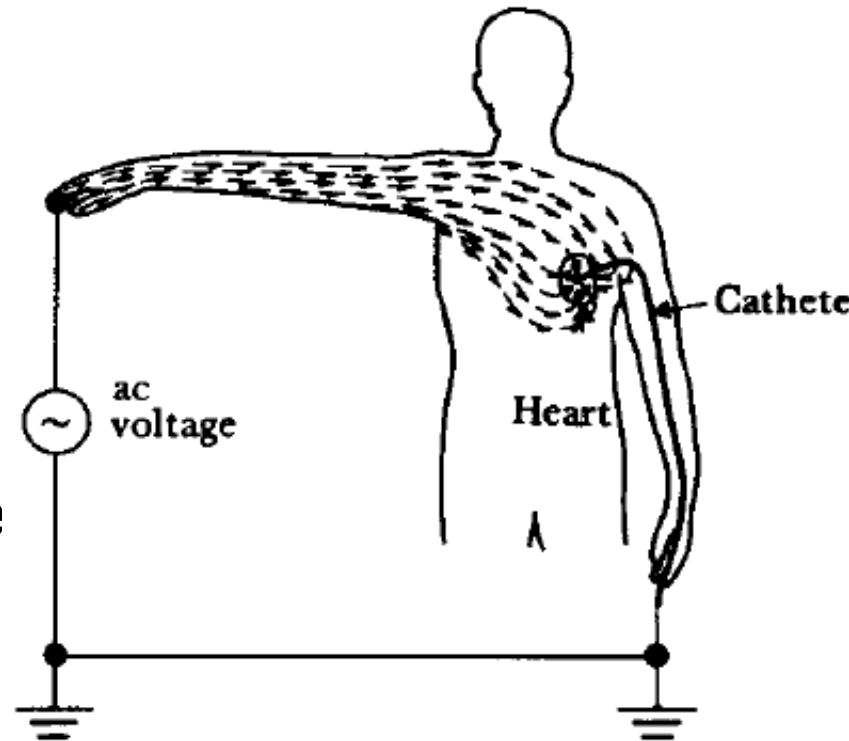
---

- Resistive heating causes burns, usually on the skin at the entry points, because skin resistance is high
- The brain and other nervous tissue lose all functional excitability when high currents pass through them.
- Excessive currents may stimulate muscular contractions that are strong enough to pull the muscle attachment away from the bone.



# Microshocks

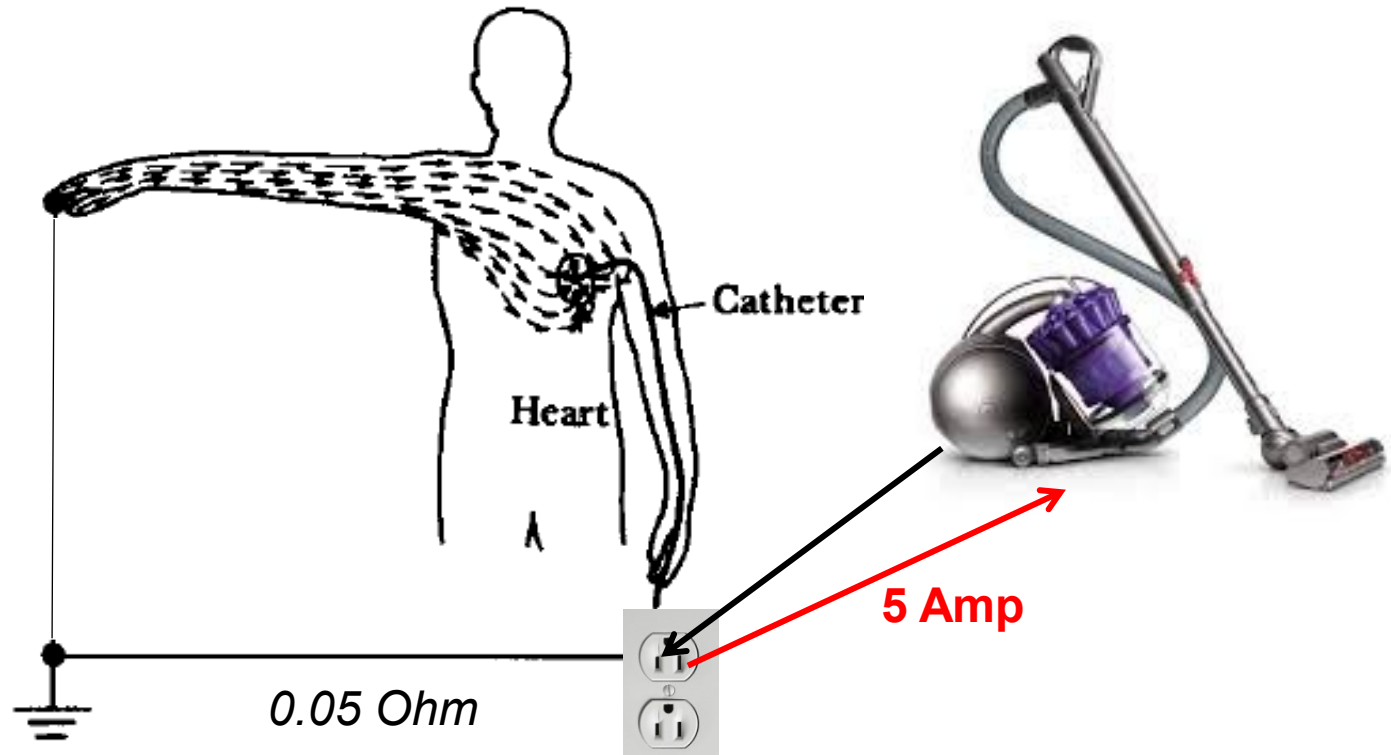
- Patients are particularly vulnerable to electric shock when invasive devices are placed in direct contact with cardiac muscle.
- If a device provides a conductive path to the heart that is insulated except at the heart, very small currents can induce fibrillation.



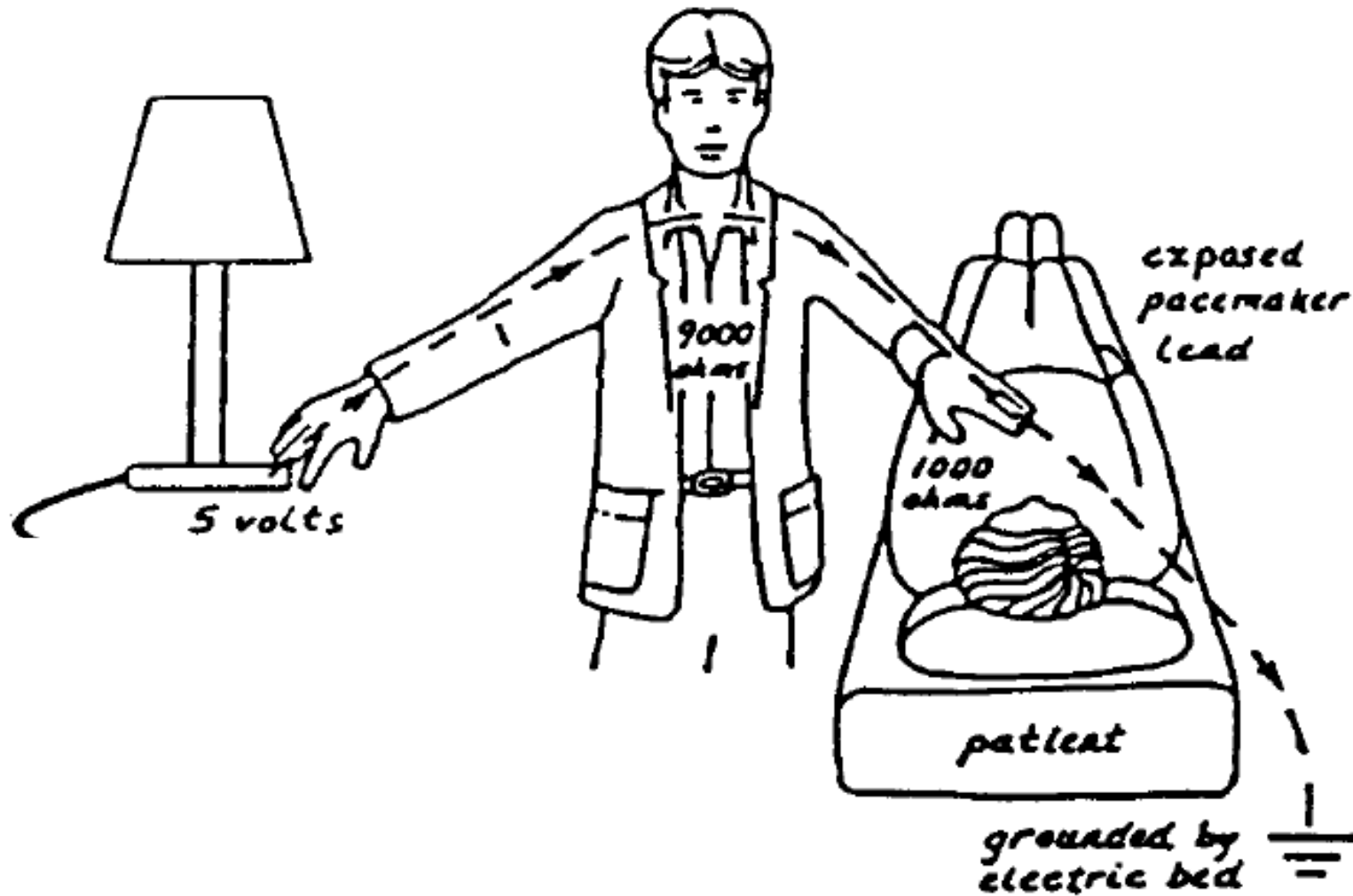


# Microshocks

- Human-heart fibrillation accidents (with an intra-cardiac catheter) indicate that microshock currents ranging from 80 to 600  $\mu\text{A}$  can cause fibrillation.



# Microshocks



# Patient Isolation

---

- It is medically necessary for some patients to be connected to leads which carry diagnostic or therapeutic signals (electrical signals).
- To prevent hazards from happening, we can limit the maximum current that can flow to a patient by using
  - High-Impedance leads
  - Current Limiters
  - Signal-Isolation Devices